Study of two-lens system with the method of Newton's lens formula

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Abstract

A study of two-lens system is presented using the Newton's lens formula. Two convex lenses of equal focal length are positioned in variable distances d and their principal focal length is calculated using the Newton's method. Initially, the equivalent optical power of the two-lens system is derived. This optical power is then plotted as a function of the distance d. The related graph provides the information for the optical power of each lens. Eventually, the focal length of one convex lens is depicted. This method can be applied in Physics Laboratories (Geometrical Optics) for undergraduate students that want to study the behavior of a two-lens system.

Keywords

Two-lens System, Focal Length, Newton's Lens Formula, Geometrical Optics, Optical Power.

Introduction

The Newton's method is widely used in Optics experiments for educational and research applications. It is well known that the focal length of a lens can be easily derived with various calculations. One approach is using Newton's lens equation. Newton's formula states that the product of the distances of two conjugate points from the respective focal point of a lens is equal to the square of its focal length.

In a recent work, a technique for the focal length measurement of lenses applying Newton's method was followed, using a cyclic path optical configuration CPOC and a wedge shear plate WSP [1]. In this work two point sources of orthogonal linearly polarized light that have a known longitudinal separation between them were generated using CPOC. Results were obtained for a positive doublet lens of focal length of 500.0 mm. Furthermore, the Newton's method was applied for the measurement of focal lengths of convex lenses by another research group [2]. They used this method because one of its advantages is that it does not require the location of the principal points of the test lens for the measurement of focal length. In this paper some modifications in this method have been presented and its modified version was applied to find out the focal length of a convex lens with better accuracies. The suggested improvements are, the use of a sufficiently illuminated target in place of cross-slit, and the travelling microscopes in place of screen.

The current work concerns the application of Newton's law to a two-lens system, in particular to a two-lens system with lenses of equal focal length. This study extends the knowledge of a traditional method in Optics to a more modern and innovative approach of a system with lenses.

Methods

If s_1 is the distance between the object and the optical center of the lens O and s_2 is the distance between the image and the optical center of the lens O, then the normal lens formula is the following equation (where f is the focal length of the convex lens) [3-6]:

$$\frac{1}{s_1}+\frac{1}{s_2}=\frac{1}{f} \tag{1}$$

If x_1 is the distance between the object and the focus F' on the object side (figure 1) and x_2 is the distance between the image and the focus F on the image side, then it is:

$$x_1 = s_1 - f \Rightarrow s_1 = x_1 + f \tag{2}$$

and

$$x_2 = s_2 - f \Rightarrow s_2 = x_2 + f \tag{3}$$

Substituting s_1 and s_2 in relation 1, one can calculate Newton's lens formula:

$$x_1 \cdot x_2 = f^2 \tag{4}$$

The focal length of a two-lens system f_s, separated by a distance d is given from the relation, where f_1 and f_2 the focal lengths of each lens respectively:

$$f_s = \frac{f_1 f_2}{f_1 + f_2 - d} \tag{5}$$

Thus, the optical power of the two-lens system P_s can be derived as below:

$$P_s = P_1 + P_2 - d \cdot P_1 \cdot P_2 \tag{6}$$

In our case the two lenses have equal focal length $f_1 = f_2 = f$ and equations 5 and 6 have the form:

$$f_s = \frac{f^2}{2f - d} \tag{7}$$

$$P_s = 2P - d \cdot P^2 \tag{8}$$

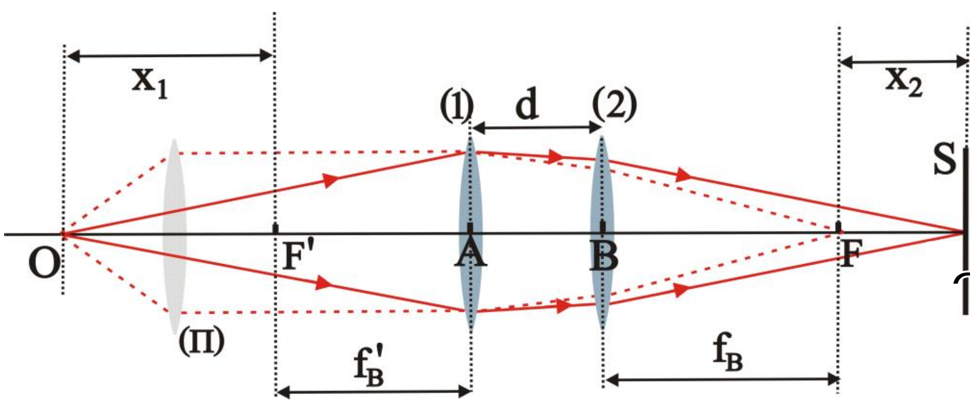

Figure 1. Two-lens system (1) and (2) separated by a distance d.

Experimental set up

In the experimental apparatus the point object is a lamp positioned on the left part of the optical bench as shown in figure 2. A collimating lens (II) in figure 2, of 0.150 m focal length is positioned after the object (first photo). Two lenses of equal focal length 0.100 m (1) and (2) in figure 2 are placed after the collimating lens, with a distance d between each other. An observation screen is positioned after the two-lens system in order to observe the focused image.
The collimating lens is not applied at the second photo of figure 3, as the measurements are followed without it.

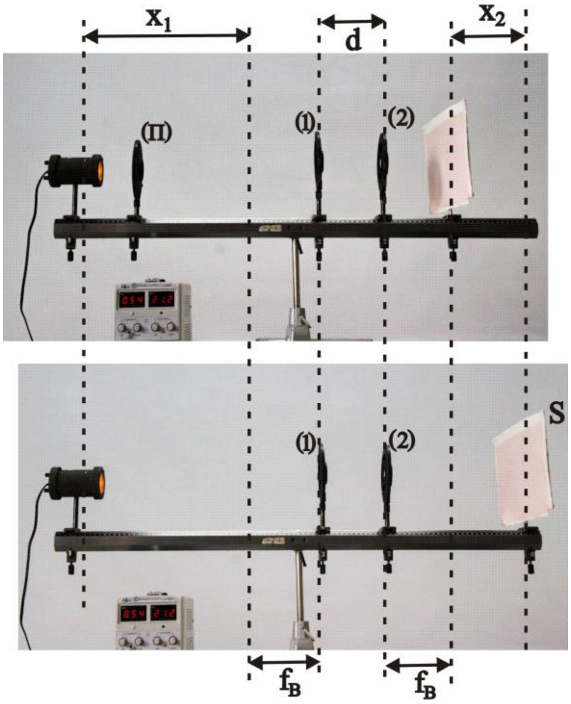

Figure 2. The experimental set up with and without the collimating lens.

Results

At the first part of the experiment the collimating lens is placed at its focal plane so that a parallel beam is maintained after it and before the two-lens system [7]. Initially, the two lenses are positioned at a distance d of 4 cm. The beam is focused at point F (figure 1). The distance BF or f_B as shown in figure 1 is the back focal length.
At the second part of the experiment the collimating lens has been removed and the beam is focused at point G (figure 1). Then x_1 = BG-BF. Due to symmetry (because the two lenses are of equal focal length) it is AF' = BF. If AO is the distance between the object and the first lens (1) as shown in figure 1, it is then x_2 = AO-AF'. Therefore, the Newton's lens formula is:

$$x_1 \cdot x_2 = f_s^2 \tag{8}$$

where f_s is the equivalent focal length of the two-lens system and this focal length can be derived as the square root of the product of x_1 and x_2. Eventually, the optical power for the specific distance d of the two-lens system is then calculated:

$$P_s = \frac{1}{f_s} \tag{9}$$

The same procedure (with and without the collimating lens) has been applied for distances d of 5 cm, 6 cm and 7 cm. The experimental results are shown in table 1.

Table 1. The distance between the two-lens system and the resulting equivalent optical power.

d (m)	x_1 (m)	x_2 (m)	f_s (m)	$P_s=1/f_s$(Dpt)
0.04	0.023	0.174	0.0632	15.80
0.05	0.025	0.170	0.0652	15.34
0.06	0.030	0.175	0.0725	13.79
0.07	0.025	0.220	0.0740	13.51

At the last part of the experiment, the diagram of the optical power as a function of the distance d, $P_s = f(d)$, provides the information of the focal length of each lens (figure 3). This diagram is a line and has the form of equation 10, which is the linear relation of equation 11.

$$P_s = 2P - d \cdot P^2 \tag{10}$$

$$y = m - kx \tag{11}$$

The intersection point of the line with y-axis, m, is equal to $2P$. Therefore the optical power of the single lens is:

$$m = 2P \Rightarrow \mathrm{P} = \frac{m}{2} \tag{12}$$

In addition, the slope of the line k can be derived from the graph and is equal to P^2.

$$k = P^2 \Rightarrow \mathrm{P} = \sqrt{\mathrm{k}} \tag{13}$$

Assuming P_1 is the lens optical power from equation 12 and P_2 the lens optical power from equation 13, then the average optical power of the single lens is $P = (P_1+P_2)/2$. This average optical power P provides the focal length of each lens just by using the equation 14:

$$\mathrm{P} = \frac{1}{\mathrm{f}} \tag{14}$$

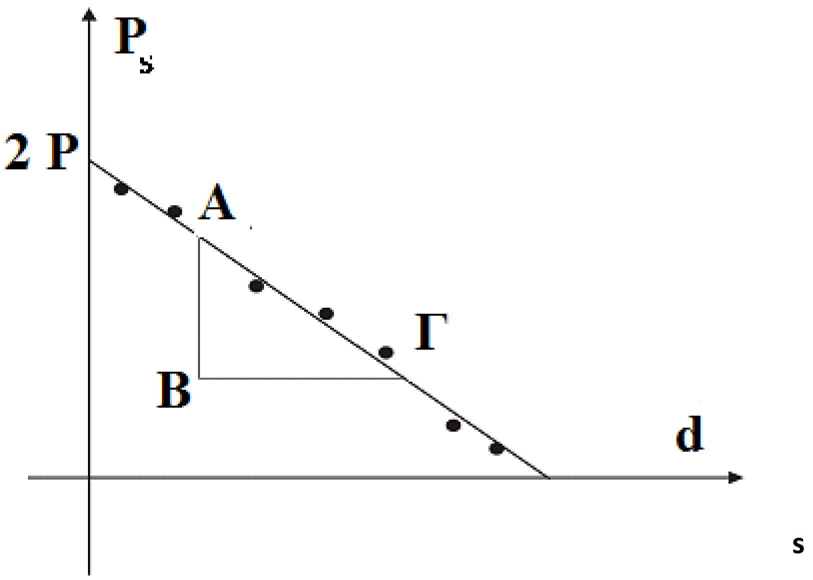

Figure 3. The equivalent optical power of the two-lens system as a function of the distance d.

The diagram is plotted and is shown in figure 4. The equation has the form

$$y = 19.241 - 84.2x \qquad (R^2 = 0.9252) \tag{15}$$

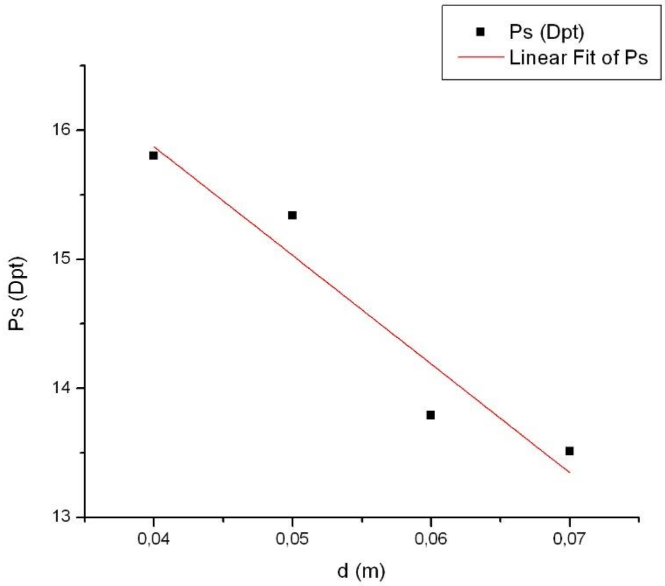

Figure 4. The experimental results of the optical power as a function of the distance.

Discussion

After the relevant calculations the optical power is $P_1 = 9.62$ Dpt and $P_2 = 9.18$ Dpt, thus the average optical power of each lens is P = 9.40 Dpt. The focal length of each lens is then derived and it is f = 0.106 m.

The results are in a good agreement with the catalogue value of the lens. The catalogue value of the lens is 0.100 m. The calculation of the focal length of each lens has a 6% variation from the catalogue value, just by applying the Newton's lens formula and using the graph of the equivalent power of the two-lens system as a function of the distance between the two lenses. This sort of experiment can be easily presented in an Optics laboratory and it is extending the knowledge of a single lens to a two-lens system, separated by a variable distance. Eventually, demonstrators don't need extra equipment to set the apparatus. The required components can be easily found in an Optics lab.

Conclusion

A two-lens system is studied using the Newton's lens equation. The two lenses have equal focal length f = 0.100 m. Their distance d has been varied and four positions have been used during the experiment. Using the Newton's lens formula one can calculate the focal length of the two-lens system. Their equivalent optical power is then derived. A graph of the equivalent two-lens optical power being on the y-axis and their distance d on the x-axis is then plotted. The interception of the line with the y-axis and the slope of the diagram give information about the optical power of the single lens. Their average optical power is derived and eventually the focal length of the single lens is calculated.

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