Software in visual basic for mathematical simulation of intensity of average monthly solar radiation on a tilted flat surface and optimal tilt angle on a flat solar collector

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Abstract

Purpose of specific study is computational simulation using software which has been developed in visual basic programming environment and calculates the average monthly intensity of solar radiation that receives a tilted surface photovoltaic or solar collector with certain tilt angle and optimal tilt angle accordance with which there are the maximum solar gains. In the first step the mathematical models that govern solar geometry and solar radiation will be described. In the second step the mathematical models are converted to programming language and the required graphical user interface is created in visual basic

Introduction

Mathematical model of solar radiation on a tilted flat surface

The ratio of incident average monthly solar radiation on a tilted surface I_T to the incident solar radiation on a horizontal surface I (Axaopoulos, 2005; Fragkiadakis, 2007) is equal to:

$$\frac{\overline{I_T}}{\overline{I}} = \left(1 - \frac{\overline{I_d}}{\overline{I}}\right)\overline{R_b} + \frac{\overline{I_d}}{\overline{I}}\left(\frac{1 + cos\beta}{2}\right) + \rho\left(\frac{1 - cos\beta}{2}\right) \quad (1)$$

Where:

- $\overline{I_T}$: Average monthly solar radiation on a tilted surface (kWh/m^2).
- $\overline{I}$: Average monthly solar radiation on a horizontal surface (kWh/m^2).
- $\overline{I_d}$: Average monthly diffuse solar radiation on a tilted surface (kWh/m^2).
- $\overline{R_b}$: The ratio of average monthly direct solar radiation on a tilted surface to the average monthly direct solar radiation on a horizontal surface.
- β: The tilt angle of surface, photovoltaic or solar collector (deg).
- ρ: The reflection of surface (dimensionless). Factor is dependent from the place which the collector is found and is selected from tables.

The ratio of I_d to the $\overline{I}$, is calculated by polynomial quadratic equation. The coefficients of equation are different and depended on the place or country of the Earth that collectors are found and we intend to determine the solar potential. For Greece (Lalas et al. 1982), the polynomial is equal to:

$$\frac{\overline{I_d}}{\overline{I}} = 1.727k^2 - 2.965k + 1.446 \quad (2)$$

k is dimensionless factor and is equal to:

$$k = \frac{\overline{I}}{\overline{I_o}} \quad (3)$$

Where $\overline{I_o}$ is the average monthly solar radiation for a surface that is found out of Earth's atmosphere limits (kWh/m^2).

$$\overline{I_o} = \frac{32.808N}{\pi}\left(1 + 0.033cos\frac{360n}{365}\right)\left(cos\varphi cos\delta sin\omega + \frac{\pi\omega}{180}sin\varphi sin\delta\right) \quad (4)$$

Where:

N: The days of month, (Table 1).
n: Characteristic day of month, (Table 1).
φ: The latitude of place (deg).
δ: Divergence of the sun (deg).
ω: Maximum hour angle in west of the sun (deg).

The value 32.808 is a result of proliferation 24 hours with solar radiation on horizontal surface out of Earth's atmosphere limits, G_o = 1.367 kW/m^2.

Divergence of helium δ is calculated by equation:

$$\delta = 23.45\sin\left[\frac{360}{365}(284 + n)\right] \quad (5)$$

Maximum hour angle of the sun is calculated by equation:

$$\omega = \arccos\left(-tan\varphi tan\delta\right) \quad (6)$$

The $\overline{R_b}$ ratio is equal to:

$$\overline{R_b} = \frac{\cos(\varphi - \beta)\, cos\delta sin\omega' + \frac{\pi\omega'}{180}sin\delta\sin(\varphi - \beta)}{cos\varphi cos\delta sin\omega + \frac{\pi\omega}{180}sin\varphi sin\delta} \quad (7)$$

Maximum hour angle ω' is equal to:

$$\omega' = \arccos\left(-\tan(\varphi - \beta)tan\delta\right) \quad (8)$$

If $\omega' >$ ω then $\omega' = \omega$, if $\omega' < \omega$ then we choose $\omega = \omega'$

With replacement of (5) to (4) (6) (7) (8), replacement of (6) to (4), replacement of (6) or (8) to (7), replacement of (3) to (2) and finally with replacement of (2) and (4) to (1), we have the average intensity of solar radiation on a tilted surface inside of Earth's atmosphere for tilted angle β.

Table 1. Information for months.

Month	Characteristic day (n)	Days of the month (N)	δ (deg)	$\bar{I}$ (kWh/m^2)
January	17	31	-20.917	51
February	16	29	-21.096	67.4
March	16	31	-21.096	111
April	15	30	-21.269	149
May	15	31	-21.269	163
June	11	30	-21.899	205
July	17	31	-20.92	212
August	16	31	-21.096	194
September	15	30	-21.269	161
October	15	31	-21.269	111
November	14	30	-21.436	75
December	10	31	-22.04	52

Mathematical model of the optimal tilt angle

The β angle is calculated by differentiation of equation (7):

$$\frac{d\overline{R_b}}{d\beta} = 0 \quad and \quad \frac{d\omega'}{d\beta} = 0 \tag{9}$$

Equation (7) becomes:

$$\frac{cos\delta[\sin(\varphi-\beta)\,sin\omega'] + sin\delta[-\omega' cos(\varphi-\beta)]}{cos\varphi cos\delta sin\omega + \omega sin\varphi sin\delta} = 0 \tag{10}$$

Equation (10) is solved by β.

With D_f: $cos\varphi cos\delta sin\omega + \omega sin\varphi sin\delta \neq 0$

$$cos\delta[\sin(\varphi-\beta)\,sin\omega'] + sin\delta[-\omega' cos(\varphi-\beta)] = 0 \Leftrightarrow$$
$$\Leftrightarrow \sin(\varphi-\beta) cos\delta\, sin\omega' - \omega' sin\delta cos(\varphi-\beta) = 0 \tag{11}$$

We set: $cos\delta\, sin\omega' = A$ and $\omega' sin\delta = B$, then (11) becomes:

$$
\begin{gathered}
A\sin(\varphi-\beta) - Bcos(\varphi-\beta) = 0 \Leftrightarrow \\
\Leftrightarrow A\sin(\varphi-\beta) = Bcos(\varphi-\beta) \Leftrightarrow \\
\Leftrightarrow A^2 sin^2(\varphi-\beta) = B^2 cos^2(\varphi-\beta) \Leftrightarrow \\
\Leftrightarrow A^2 sin^2(\varphi-\beta) = B^2 - B^2 sin^2(\varphi-\beta) \Leftrightarrow \\
\Leftrightarrow A^2 sin^2(\varphi-\beta) + B^2 sin^2(\varphi-\beta) = B^2 \Leftrightarrow \\
\Leftrightarrow (A^2+B^2) sin^2(\varphi-\beta) = B^2 \Leftrightarrow \\
\Leftrightarrow sin^2(\varphi-\beta) = \frac{B^2}{A^2+B^2} \Leftrightarrow \\
\Leftrightarrow \sin(\varphi-\beta) = \left|\sqrt{\frac{B^2}{A^2+B^2}}\right| \Leftrightarrow \\
\Leftrightarrow \beta = \varphi - \arcsin\left(\left|\sqrt{\frac{B^2}{A^2+B^2}}\right|\right)
\end{gathered}
$$

Design of user graphical interface (GUI) in visual basic

Below are reported the required graphical elements which are introduced in visual basic application. There are two sub-applications, the first for simulation and calculation of intensity of average monthly solar radiation for twelve months of the year and second for calculation of optimal angle on a tilted flat surface for twelve months of the year (Halvorson, 2005).

Table 2. Application data for simulation of average monthly solar radiation.

Variables	Graphical elements
$\overline{I_T}$ for 12 months and totally in the year (exported data)	13 labels
φ, β (imported data)	2 text boxes
Description of imported data	2 labels
Grouping of imported/exported data	2 forms
For final calculations	1 command button

Table 3. Application data for optimal tilt angle calculation.

Variables	Graphical elements
β for 12 months and average in the year (exported data)	13 labels
φ (imported data)	1 text box
Description of imported data	2 labels
Grouping of imported/exported data	2 forms
For final calculations	1 command button

Table 4. Relation between variables and labels/text boxes in the application.

Real variable	**Variable in the code**	**Labels**	**Text boxes**
φ	f	Label 1/Label 16	Text 1/Text 16
ω	w	-	-
ω'	ws	-	-
β	b	Label 2/Label 29	Text 2 Text 17 – 29
$\overline{I_T}$	It	-	Text 3/Text 15
$\overline{I}$	I	-	-
k	k	-	-
$\overline{I_o}$	Io	-	-
$\overline{R_b}$	rb	-	-
ρ	p	Label 30	Text 30

Also there are auxiliary variables w1, ws1, wh and m, m1, q, q1, Io1. All trigonometric operations in the visual basic are in radians, so is necessary the transformation parameter π/180 = 0.017 into trigonometric values (Tylee, 1998; Gaddis and Irvine 2012).

Namely, for values and numbers in radians:

$$y(\deg) = \cos\left(x(rad)\left(\frac{\pi}{180}\right)\right) \quad and \quad x(\deg) = \frac{arccos(y(rad))}{\left(\frac{\pi}{180}\right)}$$

Still, for calculations of inversion trigonometric numbers in visual basic must be introduced a code such as below:

Listing 1. Code for inversion trigonometric numbers.

```
Dim x as double

Function arccos(x As Double) As Double
arccos = Atn(-x / Sqr(-x * x + 1)) + 2 * Atn(1)
End Function

Function arcsin(x As Double) As Double
arcsin = Atn(-x / Sqr(-x * x + 1))
End Function
```

The code of application

Below it follows the code for calculation of average intensity of solar radiation for first month of year (January, n = 17, N = 31). The code in other months is exactly the same with replace of N, n and I parameters.

Listing 2. Code for average monthly solar intensity.

```
Dim c As Double
Dim f As Double
Dim w As Double
```

```
Dim ws As Double
Dim b As Double
Dim lt As Double
Dim L As Double
Dim k As Double
Dim I As Double
Dim Io As Double
Dim IdtI As Double
Dim p As Double
Dim rb As Double
Dim w1 As Double
Dim ws1 As Double
Dim wh As Double
Dim m As Double
Dim m1 As Double
Dim q As Double
Dim q1 As Double

Private Sub Command1_Click()
Const c = 0.017453292
Const I = 51
f = Text1.Text
b = Text2.Text
p = Text30.Text
w1 = -Tan(f * c) * Tan(-20.917 * c)
w = (Atn(-w1 / Sqr(-w1 * w1 + 1)) + 2 * Atn(1)) / c
ws1 = -Tan((f - b) * c) * Tan(-20.917 * c)
ws = (Atn(-ws1 / Sqr(-ws1 * ws1 + 1)) + 2 * Atn(1)) / c
If ws > w Then
wh = w
Else
wh = ws
End If
m = Cos((f - b) * c) * Cos(-20.917 * c) * Sin(wh * c)
m1 = (c * wh) * Sin(-20.917 * c) * Sin((f - b) * c)
q = Cos(f * c) * Cos(-20.917 * c) * Sin(w * c)
q1 = (c * w) * Sin(f * c) * Sin(-20.917 * c)
rb = (m + m1) / (q + q1)
Const Io1 = 333.966
Io = Io1 * (q + q1)
k = I / Io
IdtI = 1.727 * (k ^ 2) - 2.965 * k + 1.446
ItI = (1 - IdtI) * rb + IdtI * ((1 + Cos(b * c)) / 2) + p * ((1 - Cos(b * c)) / 2)
It = ItI * I
Text3.Text = It
End Sub
```

This process is repeated in the code from February to December. It will be developed now the code for calculation of optimal tilt of a flat surface for maximum solar radiation gains. There are auxiliary variables into the code: A, B1, BB, BB1, BBB1.

Listing 3. Code for calculation of optimal tilt angle.

```
Dim A As Double
Dim B1 As Double
Dim BB As Double
Dim BB1 As Double
Dim BBB1 As Double

Private Sub Command2_Click()
Const c = 0.017453292
f = Text16.Text
w1 = -Tan(f * c) * Tan(-20.917 * c)
```

```
w = (Atn(-w1 / Sqr(-w1 * w1 + 1)) + 2 * Atn(1)) / c
A = Cos(-20.917 * c) * Sin(w * c)
B1 = w * Sin(-20.917 * c)
BB = Sqr((B1 ^ 2) / (A ^ 2 + B1 ^ 2))
BB1 = (Atn(-BB / Sqr(-BB * BB + 1))) / c
If BB1 < 0 Then
BBB1 = -BB1
Else
BBB1 = BB1
End If
b = -(f - BBB1)
Text17.Text = b
End Sub
```

Example

In the following example, the average solar radiation on a tilted surface in a place with latitude φ=38.4° and tilt β=40° will be calculated, also is calculated the optimal tilt of surface in a place with latitude φ=38.4°. Orientation is southern, reflection 20%.

Table 5. Software results.

Solar intensity (kWh/m^2)		**Optimal tilt (deg)**	
Imported data		Imported data	
Latitude place, φ (deg)	38.4	Latitude place, φ (deg)	38.4
Flat tilt, β (deg)	40		
Reflectance	0.2		
Exported data		Exported data	
January	72.18	January	49.63
February	115.81	February	49.64
March	218.89	March	49.64
April	272.42	April	49.66
May	253.36	May	49.66
June	163.99	June	49.72
July	211.06	July	49.63
August	255.28	August	49.64
September	274.31	September	49.66
October	219.40	October	49.66
November	135.01	November	49.68
December	78.76	December	49.74
Total intensity	2270.38	AVG tilt	49.665

The change curves of solar radiation with regard to the time and average optimum tilt in relation to the latitude of a place will be drawn below.

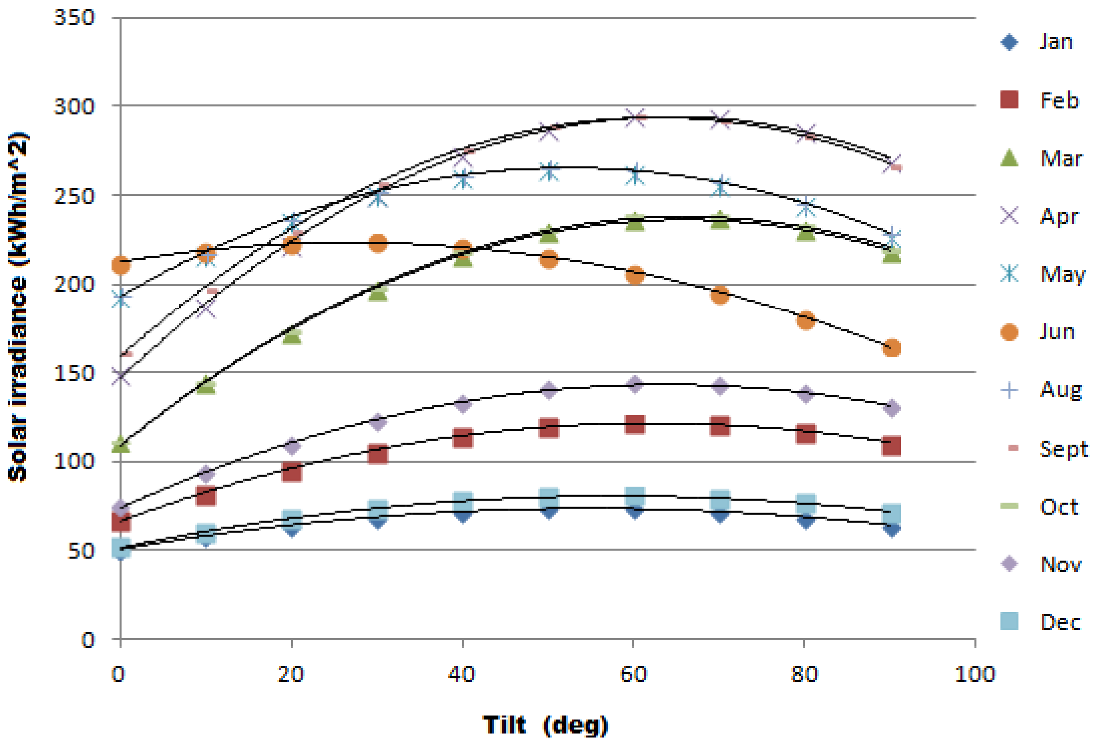

Figure 1. The change curve of average monthly solar radiation (I_T) with regard to the latitude of the place (φ).

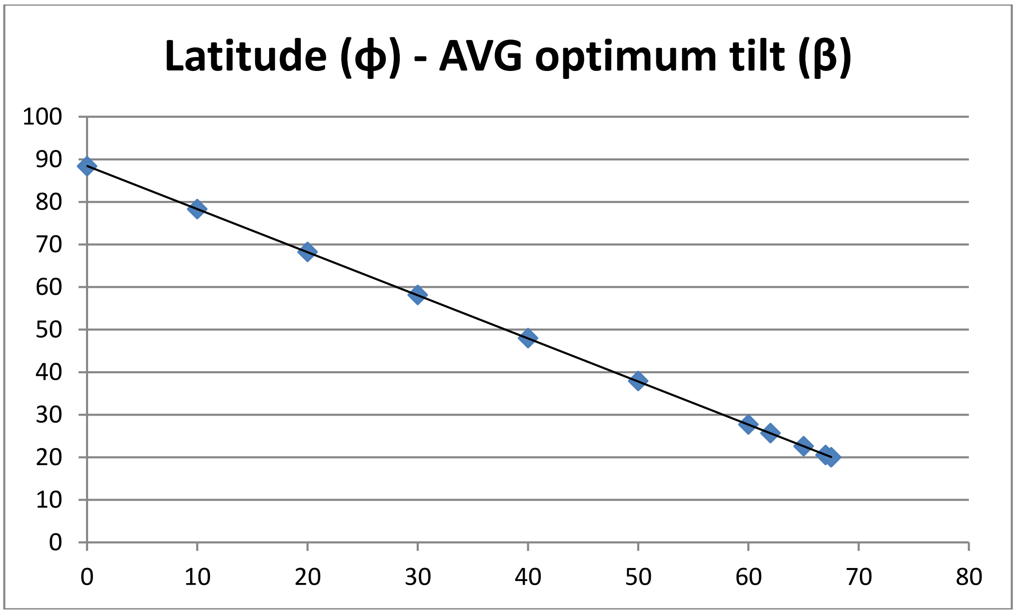

Figure 2. The change curve of average optimum tilt (β) with regard to the latitude of the place (φ).

Conclusions

The computational simulation was successfully completed. The software makes reliable calculations for latitude limits in Greece (table 5), but some errors and incorrect results for certain imported values are presented. More specifically, appears negative values of solar radiation on a tilted flat surface for φ>52° and β>1°, as the tilt angle β increases the latitude φ of the place must decreases, so there aren't negative values of solar radiation. In the application for calculation of optimal tilt angle, it's impossible to calculate the optimal tilt for latitude φ > 67.95°. These errors happen because the mathematical model that simulated is a complicated trigonometric model which has multiple domain functions. Software is recommended for educational purposes.

References

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