Blood Flow Through An Overlapping Stenosis In Catheterized Artery With Permeable Wall

Rupesh K. Srivastav[1,2], Qazi Shoeb Ahmad[2], Abdul Wadood Khan[2], Alok Mishra[1]

[1]Department of Mathematics and Computer Science
Ambalika Institute Of Management & Technology, Lucknow, India
[2]Department of Mathematics, Integral University, Lucknow, India

Corresponding Author e-mail: rupeshk.srivastav@gmail.com

Abstract
The present paper is concerned with the effect on the flow characteristics of blood due to presence of an overlapping stenosis in an inserted catheterized artery with permeable wall has been investigated. The expressions for the flow characteristics – the flow rate, the impedance (resistance to flow), the wall shear stress in stenotic region, the shear stress at the stenosis two throats and critical height of the stenosis, has been derived. A significant increase in the magnitude of the flow characteristics occurs with respect to flow parameters and catheter size.

Key words: Overlapping stenosis, catheter, permeable wall, Darcy number, slip parameter

1. Introduction

The study of the blood flow through a stenosed artery is very important because of the fact that the cause and development of many cardiovascular diseases are related to the nature of blood movement and mechanical behaviour of the blood vessel wall. The generic medical term stenosis or arteriosclerosis is the narrowing of anybody passage, tube or orifice, comes from Greek words arthero (meaning gruel or paste) and sclerosis (hardness). Although, the etiology of the initiation of stenosis is not completely understood, it is believed that the disease occur due to the deposit of cholesterol, fatty substances, cellular waste products, calcium on the arterial wall and proliferation of connective tissue may be responsible for the abnormal growth in the lumen of an artery. The flow rate and the stenosis geometry are the responsible for large pressure loss across the stenosis. The insertion of a catheter into an artery forms the annular region between the catheter wall and the arterial wall. A catheter is composed of polyster based thermoplastic polyurethane, medical grade polyvinyl choloride etc, The insertion of catheter will change the flow field and alter the haemodynamic conditions that exist in the artery before catheterization.
With the advent of the discovery that the cardiovascular disease, stenosis is closely associated with the flow conditions and other haemodynamics factors, a large number of researcher including important contributions of Mann (1938), Rodbard (1966), Young (1968, 1979), Eklof and Schwartz (1970), Young and Tsai (1973), Lee (1974), Mcdonald (1979), Ahmad and Giddens (1983), Ponalagusami (1986), Back (1994) and Back et al. (1996) studied the mean flow resistance increase during coronary artery catheterization in normal as well as stenosed arteries.

Srivastava and Srivastava (2009) have presented a brief review of the literature on artery catheterization with and without stenosis. Layek et al. (2009) investigated the effect of an overlapping stenosis on flow characteristics considering the pressure variation in both the radial and axial direction of the arterial segment under consideration. Recently Srivastav V. P. et al. investigate the effect on flow characteristics of blood due to the presence of overlapping stenosis in an artery assuming that the flowing blood represented by Newtonian fluid.
The plasma membrane is a thin, elastic membrane around the cell which usually allows the movement of small irons and molecules of various substances through it. This nature of plasma membrane is termed as permeability. The flow in the permeable boundary is described by Darcy law which states that the rate at which a fluid flows through a permeable substance per unit area is equal to the permeability times the pressure drop per unit length of flow, divided by the viscosity of fluid.
The research reported here is devoted to the study of blood flow through an overlapping stenosis in catheterized artery with permeable wall assuming that the flowing blood is represented by a Newtonian fluid.

2. Formulation of the Problem

In this paper the flow of blood through an inserted catheterized artery of circular cross-section having axisymmetric overlapping stenosis specified at the position as figure: 1 is considered. The geometry of the stenosis which is assumed to be manifested in the arterial wall segmented is described (Chakravarti and Mandal, 1994; Layek et al., 2009; Srivastava et al., 2010) as:

$$\frac{R(z)}{R_0}=\begin{cases}1-\dfrac{3}{2}\dfrac{\delta}{R_0L_0^4}\left[11(z-d)L_0^3-47(z-d)^2L_0^2+72(z-d)^3L_0-36(z-d)^4\right], & d\le z\le d+L_0,\\ 1, & \text{otherwise}\end{cases}\qquad (1)$$

where $(R(z), R_0)$ are the radius of the tube (with, without) stenosis, L_0 is the stenosis length, d indicates the stenosis location, δ is the maximum height of the stenosis into the lumen, appears the location: $z=d+L_0/6$ and $z=d+5L_0/6$, z being the axial coordinate. The height of the stenosis at $z=d+L_0/2$, called critical height, is $3\delta/4$.

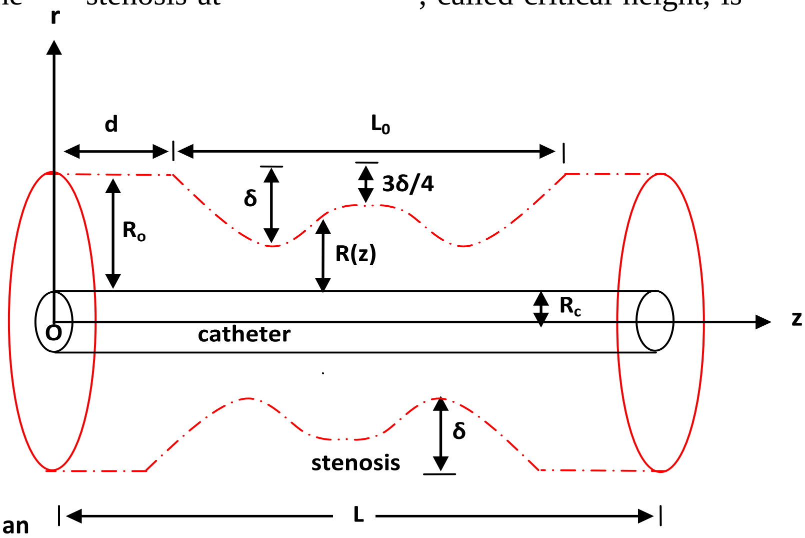

Figure: 1. The geometry of an overlapping stenosis in inserted catheterized artery with permeable wall

The equation describing the laminar, steady, one-dimensional flow in the case of a mild stenosis $(\delta << R_0)$ are expressed (Young, 1968; Srivastava et al., 2009) as

$$\frac{dp}{dz} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right) \tag{2}$$

where u is the axial velocity, μ is the fluid viscosity, r is the radial co-ordinate and p is the pressure.
The condition that are specified at the artery wall and the interface for present study may now be stated (Beavers and Joseph, 1967; Srivastava et al., 2012) as

$$u = 0, \quad \text{at } r = R_c \tag{3}$$

$$u = u_B \text{ and } \frac{\partial u}{\partial r} = \frac{\alpha}{\sqrt{k}}\left(u_B - u_{porous}\right), \quad \text{at } r = R(z) \tag{4}$$

where $u_{porous} = \frac{-k}{\mu}\frac{dp}{dz}$, u_{porous} is the velocity in the permeable boundary, u_B is the slip velocity, k is the Darcy number and α (called slip parameter) is a dimensionless quantity depending on the material parameters which characterize the structure of the permeable material within the boundary region.

3. Analysis

Using boundary conditions (3) and (4), the expression for the velocity obtained as solution of equation (2), is given as

$$u = \frac{-1}{4\mu}\frac{dp}{dz}\left[R^2 - r^2 + \frac{\left(R^2 - R_c^2\right)}{\log R/R_c}\log\frac{r}{R}\right] + \left[\frac{\log r/R}{\log R/R_c} + 1\right]u_B \tag{5}$$

Also the slip velocity, u_B is determined as

$$u_B = \frac{-1}{4\mu}\frac{dp}{dz}\left[-2R + \frac{\left(R^2 - R_c^2\right)}{R\log\left(R/R_c\right)} + 4\alpha\sqrt{k}\right]\frac{1}{\left\{\alpha/\sqrt{k} - 1/R\log\left(R/R_c\right)\right\}} \tag{6}$$

An application of equation (6) into (5), yields

$$u = \frac{-1}{4\mu}\frac{dp}{dz}\left[\begin{array}{c}\left\{R^2 - r^2 + \frac{\left(R^2 - R_c^2\right)}{\log R/R_c}\log\frac{r}{R}\right\} + \left\{\frac{\log r/R}{\log R/R_c} + 1\right\} \\ \left\{\left(-2R + \frac{\left(R^2 - R_c^2\right)}{R\log\left(R/R_c\right)} + 4\alpha\sqrt{k}\right)\Big/\left(\frac{\alpha}{\sqrt{k}} - \frac{1}{R\log\left(R/R_c\right)}\right)\right\}\end{array}\right] \tag{7}$$

The volumetric flow rate, Q is now calculated as

$$Q = 2\pi \int_{R_c}^{R} urdr$$

$$Q = \frac{-\pi R_0^4}{8\mu}\frac{dp}{dz}\left[\begin{array}{r}\left(\frac{R}{R_0}\right)^4\left(1-\frac{1}{\log(R/R_c)}\right)+\left(\frac{R}{R_0}\right)^2\left(\frac{2\varepsilon^2}{\log(R/R_c)}-\frac{\beta}{R_0^2\log(R/R_c)}-\frac{2\beta}{R_0^2}\right)\\ -\frac{\varepsilon^2}{\log(R/R_c)}\left(\varepsilon^2-\frac{\beta}{R_0^2}\right)-\varepsilon^4+\left(\frac{4\beta\varepsilon^2}{R_0^2}\right)\end{array}\right]$$

where $\varepsilon = \frac{R_c}{R_0}$, and (8)

$$\beta = \left(-2R + \frac{\left(R^2 - R_c^2\right)}{R\log(R/R_c)} + 4\alpha\sqrt{k}\right) \Big/ \left(\frac{\alpha}{\sqrt{k}} - \frac{1}{R\log(R/R_c)}\right)$$

From equation (8), one now obtains

$$\frac{dp}{dz} = -\frac{8\mu Q}{\pi R_0^4}\phi(z), \text{ where } \phi(z) = \frac{1}{h(z)} \tag{9}$$

and with

$$h(z) = \left[\begin{array}{r}\left(\frac{R}{R_0}\right)^4\left(1-\frac{1}{\log(R/R_c)}\right)+\left(\frac{R}{R_0}\right)^2\left(\frac{2\varepsilon^2}{\log(R/R_c)}-\frac{\beta}{R_0^2\log(R/R_c)}-\frac{2\beta}{R_0^2}\right)\\ -\frac{\varepsilon^2}{\log(R/R_c)}\left(\varepsilon^2-\frac{\beta}{R_0^2}\right)-\varepsilon^4+\left(\frac{4\beta\varepsilon^2}{R_0^2}\right)\end{array}\right]$$

The pressure drop, $\Delta p(= p \text{ at } z = 0, \text{-}p \text{ at } z = L)$ across the stenosis in the tube of length, L is obtained as

$$\Delta p = \int_0^L \left(-\frac{dp}{dz}\right)dz = \frac{8\mu Q}{\pi R_0^4}\psi \tag{10}$$

$$\text{where } \psi = \int_0^d [\phi(z)]_{R/R=1} dz + \int_d^{d+L_0} [\phi(z)]_{R/R_0 \text{ from}(1)} dz + \int_{d+L_0}^{L} [\phi(z)]_{R/R=1} dz$$

The first and the third integrals in the expression for obtained above are straight forward whereas the analytical evaluation of second integrals are almost a formidable task and therefore shall be evaluated numerically.

The flow resistance (resistive impedance), $\overline{\lambda}$, the wall shear stress, $\overline{\tau_w}$, of the stenosis are now calculated as

$$\bar{\lambda} = \frac{\Delta p}{Q} = \frac{8\mu}{\pi R_0^4}\left[\frac{L-L_0}{\Omega} + \int_d^{d+L_0} [\phi(z)]_{R/R_0 \; from \; (1)} \, dz\right] \tag{11}$$

$$\bar{\tau}_w = -\frac{R}{2}\frac{dp}{dz} = \frac{4\mu Q}{\pi R_0^3}\left(\frac{R}{R_0}\right)\phi(z) \tag{12}$$

Following now the reports of Srivastava et al, 2009, one derives the expressions for the impedance, λ , the wall shear stress, τ_w, of the stenosis in their non dimensional form as

$$\lambda = \eta\left[\frac{1-L_0/L}{\Omega} + \frac{1}{L}\int_d^{d+L_0} [\phi(z)]_{R/R_0 \; from(1)} dz\right] \tag{13}$$

$$\tau_w = \eta \frac{(R/R_0)}{\left[\left(\frac{R}{R_0}\right)^4\left(1-\frac{1}{\log(R/R_0)}\right) + \left(\frac{R}{R_0}\right)\left(\frac{2\varepsilon^2}{\log(R/R_0\varepsilon)} - \frac{\beta}{R_0^2\log(R/R_0\varepsilon)} - \frac{2\beta}{R_0^2}\right) - \frac{\varepsilon^2}{\log(R/R_0\varepsilon)}\left(\varepsilon^2 - \frac{\beta}{R_0^2}\right) - \varepsilon^4 + 4\varepsilon^2\frac{\beta}{R_0^2}\right]} \tag{14}$$

$$\tau_s = [\tau_w]_{R/R_0 = 1-5\delta/4R_0},$$

$$\tau_c = [\tau_w]_{R/R_0 = 1-3\delta/4R_0},$$

where, $$\frac{\beta}{R_0^2} = \frac{\left[-2\left(\frac{R}{R_0}\right) + \frac{(R/R_0)^2 - (R_c/R_0)^2}{(R/R_0)\log(R/R\varepsilon)} + \frac{4\alpha\sqrt{k}}{R_0}\right]}{\left[\frac{R_0\alpha}{\sqrt{k}} - \frac{1}{(R/R_0)\log(R/R_0\varepsilon)}\right]}, \tag{15}$$

$$\Omega = (1-\varepsilon^2)\left(1-\varepsilon^2+\frac{\beta}{R_0^2}\right)\frac{1}{\log\varepsilon} - (1-2\varepsilon^2)\frac{2\beta}{R_0^2} + 1 - \varepsilon^4,$$

$$\lambda = \bar{\lambda}/\lambda_0, \; (\tau_w, \tau_s, \tau_c) = (\tau_w, \tau_s, \tau_c)/\tau_0, \; \lambda_0 = \frac{8\mu L}{\pi\eta R_0^4}, \; \tau_0 = \frac{4\mu Q}{\pi\eta R_0^3} \tag{16}$$

are respectively the resistive impedance and wall shear stress for an uncatheterized normal artery (no stenosis).
In absence of the catheter (i.e., under the limit $\varepsilon \to 0$), one derives the expressions for λ, τ_w, respectively through an overlapping stenosis with permeable wall as

$$\lambda = \eta \left[\frac{1 - L_0/L}{\Omega} + \frac{1}{L} \int_{d}^{d+L_0} \frac{dz}{(R/R_0)^2 \left\{ (R/R_0)^2 + \frac{4\sqrt{k}}{\alpha R_0^2} \left(R - 2\alpha\sqrt{k} \right) \right\}} \right] \tag{17}$$

$$\tau_w = \frac{\eta}{(R/R_0) \left\{ (R/R_0)^2 + \frac{4\sqrt{k}}{\alpha R_0^2} \left(R - 2\alpha\sqrt{k} \right) \right\}} \tag{18}$$

4. Numerical Results and Discussion

The development of a stenosis in an artery can obviously create many serious problems and in general disrupt the normal function of the circulatory system. In order to observe the quantitative effects of the various parameters involved in the present analysis, computer codes are developed to evaluate the analytical results obtained in equations (13)-(14) for dimensionless resistance to flow, λ the wall shear stress , τ_w in the stenotic region in a tube of radius 0.01*cm* for various parameter values: *d*=0; $L_0(cm)$ =1; *L (cm)*= 1, 2, 5; α =0.1, 0.2, 0.3, 0.4, 0.5; $\sqrt{k}$ =0.1, 0.2, 0.3, 0.4, 0.5; δ/R_0 =0, 0.05, 0.10, 0.15, 0.20. It is to note that fluid considered here is Newtonian.

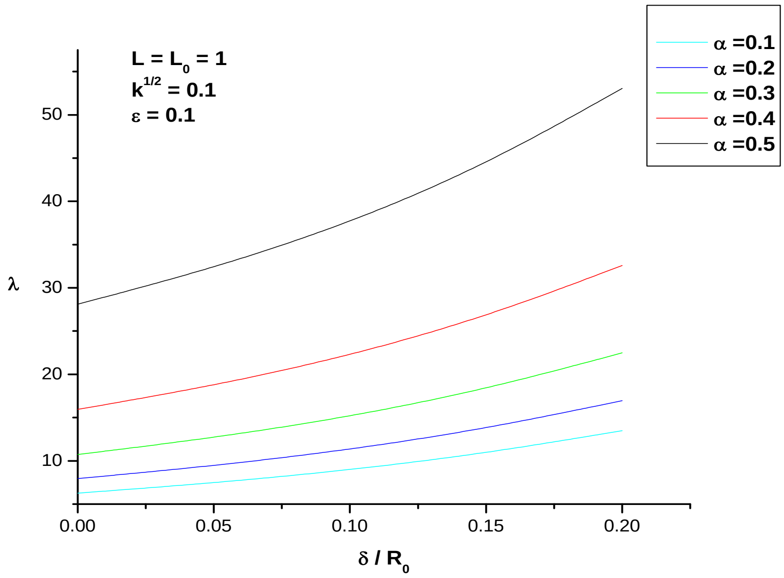

Figure:2. Impedance, λ versus stenosis height, δ / R_0 for different α.

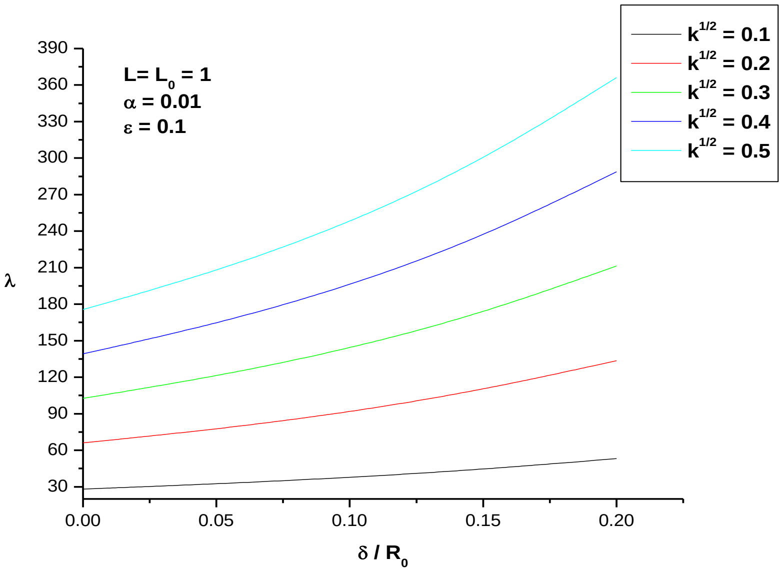

Figure:3. Impedance, λ versus stenosis height, δ / R_0 for different k.

One notices that the blood flow characteristic, impedance, λ increases with increasing value of slip parameter, α, and also increases with stenosis height, δ/R_0, for given value of Darcy number, k, and catheter size, ε, (Figure: 2).

For a given slip parameter, α, and catheter size , ε, the impedance (flow resistance) , λ, increases with increasing value of Darcy number, k, and also increases with stenosis height, δ/R_0 (Figure: 3).

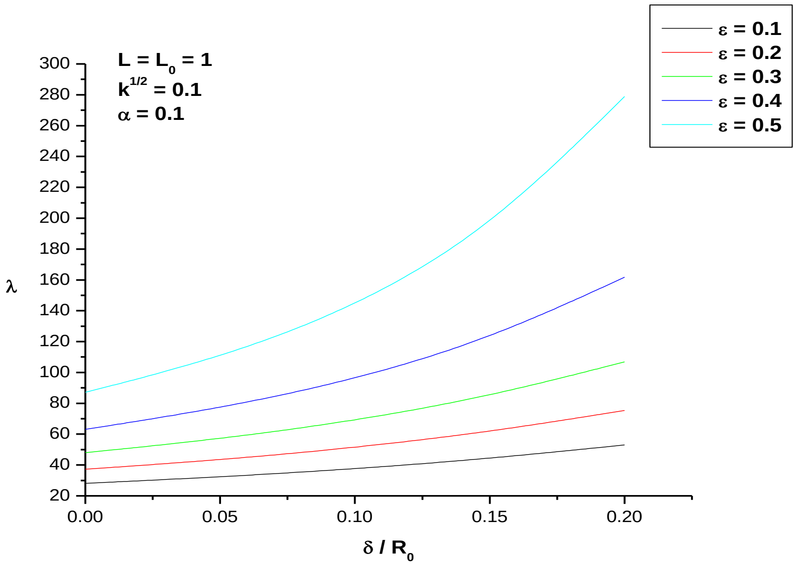

Figure:4. Impedance, λ versus stenosis height, δ / R_0 for different ε.

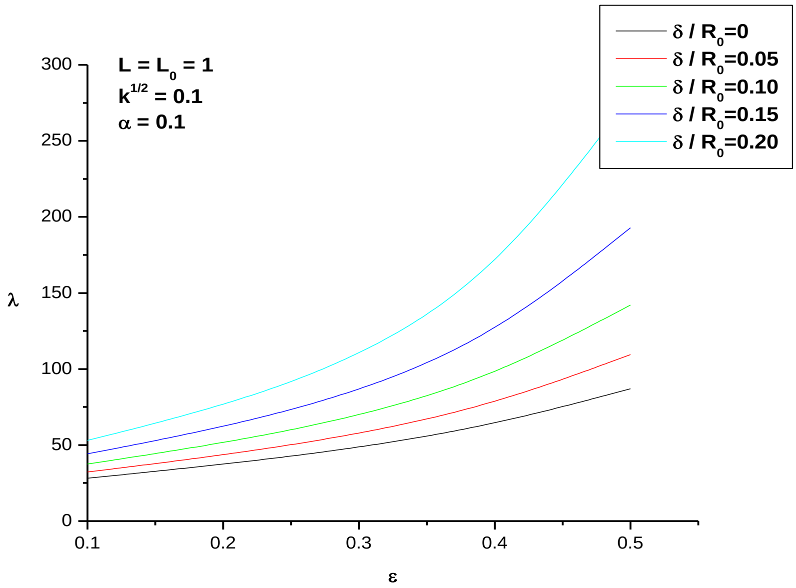

Figure:5. Impedance, λ versus catheter radius, ε for different δ / R_0.

It is observed that the presence of the catheter causes increase in the magnitude of impedance , λ, in addition to that has occurred due to the presence of the stenosis (Figure: 4, 5).

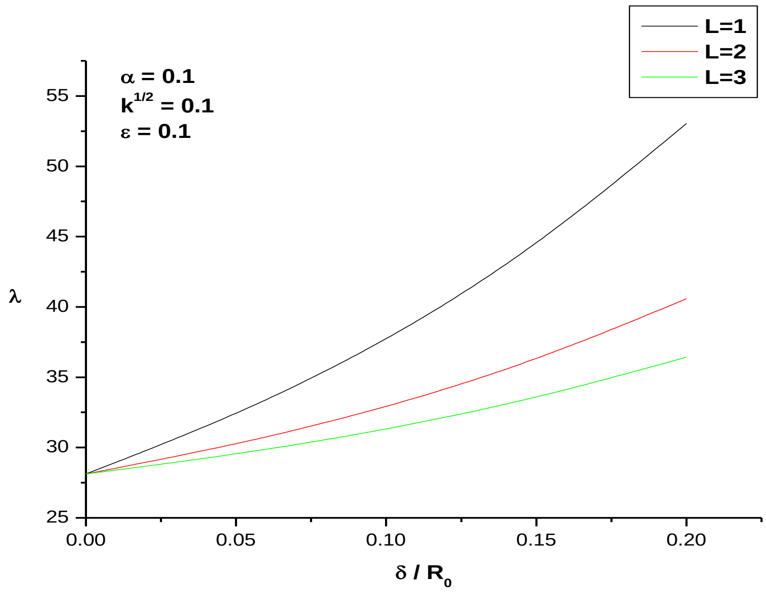

Figure:6. Impedance, λ versus stenosis height, δ / R_0 for different L.

For a given Darcy number,k , the catheter size, ε , the impedance, λ , decreases with increasing tube length, L , increases with stenosis size (Figure: 6, 7)

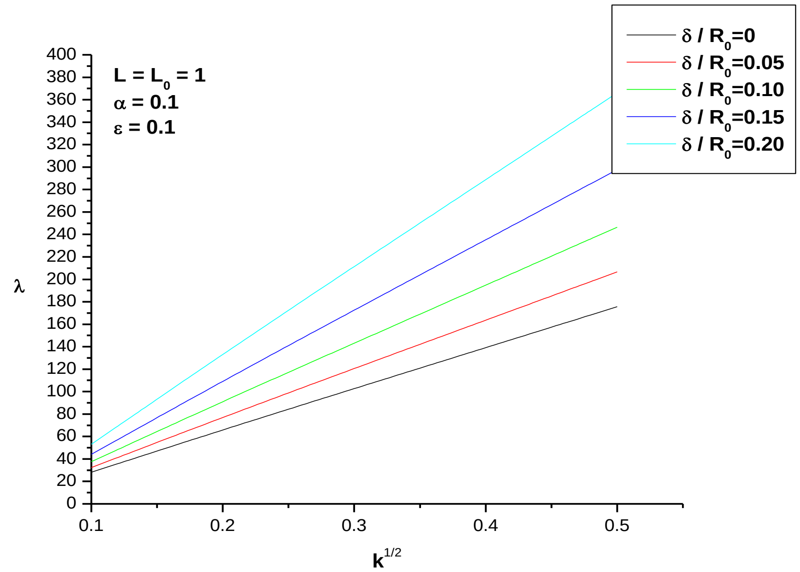

Figure:7. Impedance, λ versus Darcy number, k for different δ / R_0

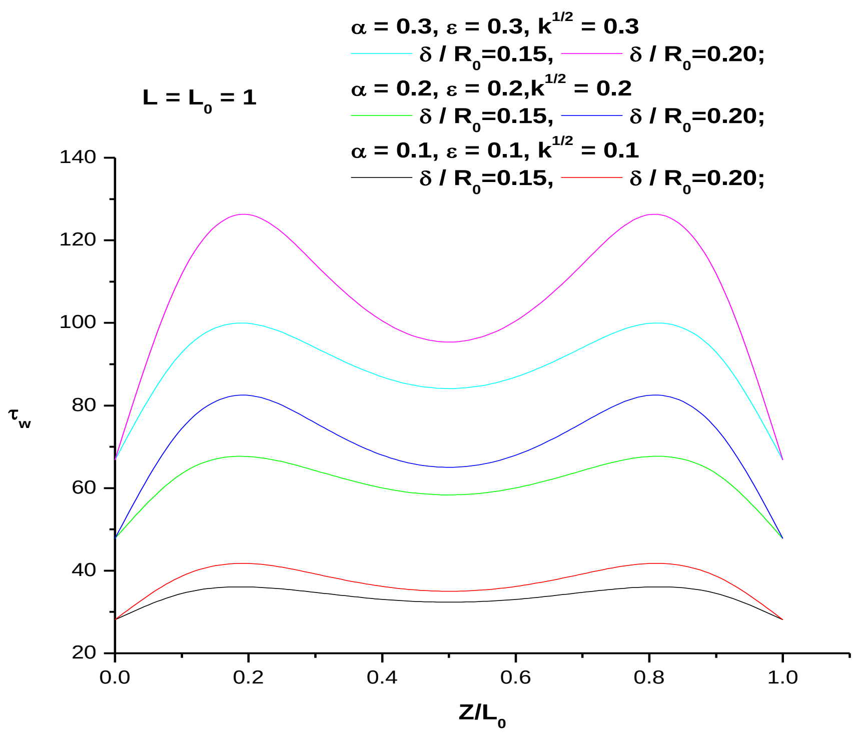

Figure:8. Wall shear stress, impedance, τ_w in stenotic region for different α, ε , $k^{1/2}$ and δ / R_0.

Figure: 8, shows that the wall shear stress in stenotic region, τ_w increases with increasing value of slip parameter, α, catheter radius, ε, Darcy number, k, and increases rapidly with the axial distance, z/L_0 in upstream of the stenosis first throat and attains its peak value at the first throat (i.e. at $z/L_0 = 1/6$). It then decreases steeply in the downstream of the first throat to its magnitude at stenosis critical height located at $z/L_0 = 1/2$. The wall shear stress, τ_w further increases steeply in the upstream of the stenosis second throat at $z/L_0 = 5/6$ and attains its peak value at second throat. The flow characteristic, τ_w then decreases in the downstream of the stenosis second throat and attains its approached magnitude (at $z/L_0 = 0$) at end point of constriction profile at $z/L_0 = 1$.

Conclusions

In this paper, we have studied the effect on flow characteristics of a Newtonian fluid in an inserted catheterized stenosed artery in a circular tube with permeable wall. The impedance increases with various parameter as well as catheter size. The wall shear stress possesses characteristics similar to that of the flow resistance with respect to given parameter.

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