Simulating Key Aspects of the Game of Soccer by Use of a Mathematical Model

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Abstract

A mathematical model together with simulation data is presented for a soccer ball in flight. This type of sports ball although used in a sport of great popularity has received limited attention within the research community. It is concluded that a relatively simple model can be used in order to simulate key situations during a soccer game. Initial simulation results are presented for a free kick, corner kick and goal kick under various conditions. The results are consistent to previous studies and to the physical principles governing soccer ball motion in the air.

Key words: *Soccer, Simulation, Magnus Force*

Nomenclature

A	ball cross sectional area
C_d	drag coefficient
C_{mag}	Magnus force coefficient
$\mathbf{D}$	drag vector
$\mathbf{F_{mag}}$	Magnus force vector
$\mathbf{g}$	gravitational constant
k_d	drag constant
k_{mag}	Magnus force constant
m	ball mass
$\mathbf{r}$	position vector
$\mathbf{r}_0$	initial position vector
$\mathbf{u}$	velocity vector
$\mathbf{u}_0$	initial velocity vector
$\mathbf{x_0}$	initial conditions vector
ρ	air density
$\boldsymbol{\omega}$	spin vector
$\boldsymbol{\omega}_0$	initial spin vector

Introduction

Scientific interest in sports ball dynamics has existed since the time of Newton who was the first to observe that tennis balls bend when in flight and attributed this to the surrounding air. Tait$^{(1)}$ performed one of the first proper investigations on sports ball flight by examining the effect of spin on golf ball range. Thompson$^{(2)}$ also studied the dynamics of a golf ball, the aerodynamic properties of which have been subsequently studied by Davies$^{(3)}$ and Bearman & Harvey$^{(4)}$. The effect of ball properties (such as seams) on baseball and cricket ball swing has also been the focus of extensive research (for example Watts & Ferrer$^{(5)}$, Alaways & Hubbard$^{(6)}$, Mehta$^{(7)}$). The effect of tennis ball aerodynamic characteristics has been studied by Stepanek$^{(8)}$ and Haake et al$^{(9)}$.

By contrast relatively few studies have been conducted on soccer balls. Carré et al$^{(10)}$ and Bray & Kerwin$^{(11)}$ have both used trajectory data obtained from video recordings in order to model soccer ball flight, primarily focusing on the free kick set play. Griffiths et al$^{(12)}$ used a motion analysis system to obtain accurate measurements of the ball spin vector, position and velocity.

In this current study, wind tunnel data, published recently by Asai et al$^{(13)}$, is inspected in order to model the aerodynamic forces that drive the motion of a soccer ball in flight.

The model

Wesson$^{(14)}$ has produced a good account of the physics governing the flight of a soccer ball. He assumes, as do Bray & Kerwin$^{(11)}$ and Carré et al$^{(12)}$, that when in the air a soccer ball is under the influence of three forces: the weight, the aerodynamic drag and the Magnus force, as shown in Fig. 1. In this particular case the ball is portrayed travelling horizontally with pure backspin and the Magnus force acts opposite to the weight.

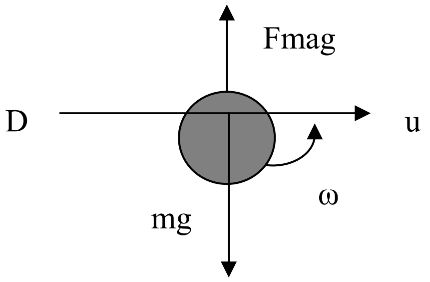

Figure 1: Forces acting on a rotating soccer ball

Assuming the weight of the ball as being constant and equal to the gravitational force, our attention is drawn to the two forces generated by the interaction with the surrounding air.

Mathematically the drag acting on the ball is derived from the formula:

$$\mathbf{D} = -\frac{1}{2}\rho A|\mathbf{u}|^2 C_d \frac{\mathbf{u}}{|\mathbf{u}|} \qquad (1)$$

The drag coefficient of sports balls has been shown (Mehta(7)), as being primarily a function of Reynolds number and sphere roughness, although Asai et al(13) have found for soccer balls that there is also a dependency on the spin rate, if the ball is rotating.
An interesting phenomenon related to the drag coefficient of spheres and thus soccer balls, is that a "drag crisis" occurs once the velocity has increased and a critical Reynolds number is reached. At this point a transition occurs from a laminar boundary layer to a turbulent boundary layer and the drag coefficient drops by a substantial margin. Asai et al(13) observed that for soccer balls this will occur at a speed of 12-15 m/s and since most powerful kicks of interest (for example free kicks) have an initial speed of 25-35 m/s, it is assumed that mainly post critical conditions are encountered.
The flow pattern of a spinning ball can be used to explain the now well understood Magnus effect, first studied by Robins and Magnus. It can be observed in Fig. 2 that on the side of the ball moving with the flow, viscous effects will carry the air further around the ball before separation occurs. The opposite is manifested on the side that is moving against the flow. The overall result is for the air to be deflected to one side, in effect pushed by the spinning ball, and in order for momentum to be conserved and equal an opposite force must be exerted by the air on the ball itself.

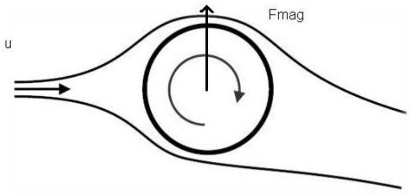

Figure 2: The Magnus effect

In mathematical terms the Magnus force is derived from the formula:

$$\mathbf{F}_{\mathbf{mag}} = \frac{1}{2}\rho A|\mathbf{u}|^2 C_{mag} \frac{\boldsymbol{\omega}\times\mathbf{u}}{|\boldsymbol{\omega}\times\mathbf{u}|} \qquad (2)$$

So for a ball kicked with pure sidespin the Magnus force will deflect the trajectory either to the left or the right depending on the direction of the spin, resulting in the, so called, "banana kick". When pure backspin is imparted the Magnus force will oppose the weight thus increasing the range as observed by Tait(1) for the case of golf balls.

Having obtained expressions for all forces driving the flight of the ball and assuming the spin vector to be constant over the duration of the flight, it is then possible to derive the equation of motion, which in vector form is:

$$\ddot{\mathbf{r}} = k_{mag}|\mathbf{u}|^2 \frac{\boldsymbol{\omega}\times\mathbf{u}}{|\boldsymbol{\omega}\times\mathbf{u}|} - k_d|\mathbf{u}|^2 \frac{\mathbf{u}}{|\mathbf{u}|} + \mathbf{g} \qquad (3)$$

Where,

$$\ddot{\mathbf{r}} = \frac{d^2\mathbf{r}}{dt^2}$$

$$k_{mag} = \frac{\rho A}{2m} C_{mag}$$

$$k_d = \frac{\rho A}{2m} C_d$$

The above equation has no closed form solution and was solved using a Runge-Kutta routine in the Matlab® software. In order to perform this, the drag and Magnus coefficients have to be determined.

In the work described in this paper the method of utilising results published from wind tunnel tests is adopted. Asai et al(13) are the first to have conducted an extensive study of the aerodynamic properties of three types of soccer balls all of which have been officially approved for use in international games. Values for the drag and Magnus coefficients were determined for various Reynolds numbers and spin parameter (a non-dimensional parameter related to the spin rate of the ball). As stated by the authors themselves “these data are valuable for calculating the ball trajectory”, a challenge taken up by the research work described here. For post critical conditions, inspection of the aforementioned data suggests that the drag coefficient reaches an approximately constant value of 0.15. Furthermore, for spin parameter values typical of game situations, the Magnus coefficient is seen to be approximately equal to the spin parameter itself. It should be noted that for greater accuracy in the results, the full set of wind tunnel data should be used. Nevertheless, as will be shown in the following sections, this simple model (of assuming the drag coefficient equal to 0.15 and the Magnus coefficient equal to spin parameter), is still capable of correctly emulating the physical principles of soccer ball flight.

Simulation results

Simulation trials have until this point focused on three of the most important soccer game set plays, that is, the free kick, the corner kick and the goal kick. In each case the solver algorithm is provided with the appropriate initial conditions vector, i.e.,

$$\mathbf{x_0} = [\mathbf{r_0} \mathbf{u}_0 \, \boldsymbol{\omega}_0]$$

At each time step the position vector and the velocity vector of the ball are computed until a set termination condition is reached (for the case of the free kick for example, this is usually taken as the point when the ball crosses the goal line).

Free kick results

For the free kick modelling it was of interest to simulate typical cases from which professional players are likely to attempt a direct goal. In order to realistically emulate the real game situation, a wall, in the form of a rectangle situated at 9.15 m from the free kick position, had to be cleared for the kick to be successful. A three dimensional view of such a kick taken from the centre line at 18.28 m (20 yards) from the goal is illustrated in Fig. 3(a). For this particular kick the initial elevation of the velocity vector is 17 deg, the initial velocity is 25 m/s and the spin imparted is pure sidespin of 7 rev/s. These values are typical of those achieved by professional players. In Fig. 3(b) the computed trajectory is presented for a free kick of the same initial conditions apart from the elevation that has been increased by 1 degree. Interestingly in this case the ball flies over the bar in a failed attempt to score. Similarly when decreasing the initial elevation by 1 degree (as seen in Fig. 3(c)), the ball fails to clear the wall.

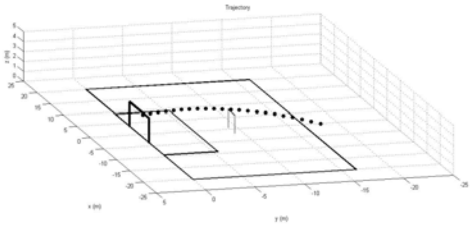

Figure 3 (a): Free kick with 17 deg elevation

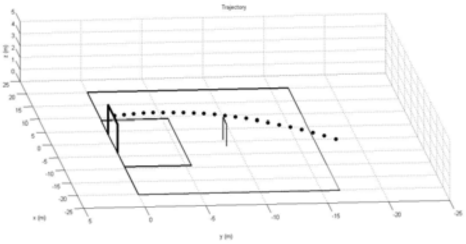

Figure 3 (b): Free kick with 18 deg elevation

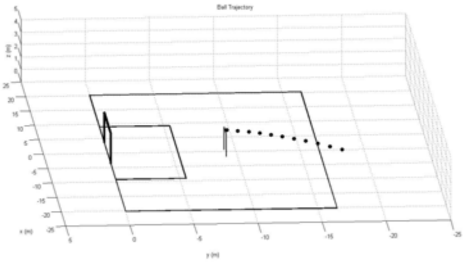

Figure 3 (c): Free kick with 16 deg elevation

In Fig. 4, the variation of the velocity components of the ball during the 17 degree elevation free kick are presented.

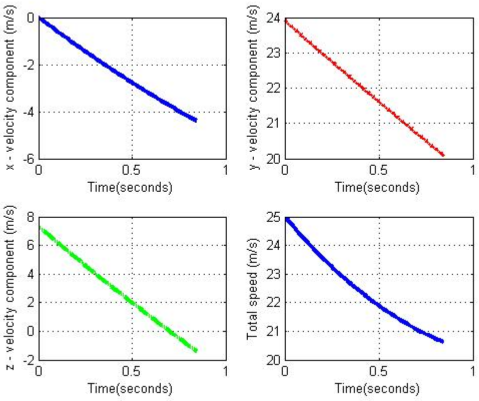

Figure 4: Velocity components for 17 deg elevation free kick

Interestingly, the total speed drops by more than 4 m/s but this flight condition, according to the results of Asai et al$^{(13)}$, is still in the post critical drag regime.
At the start of the free kick the drag and the Magnus coefficient were calculated at 0.15 and 0.19 which are typical values for those parameters. This yields initial drag and Magnus forces of approximately 2.14 N and 2.64 N. Assuming that the mass of the ball is 0.43 kg, this in turn yields acceleration values of approximately 6.14 m/s^2, close to 0.6 g in the lateral direction. For a free kick lasting for less than one second this will result in a lateral deflection of the order of 2m, often enough to deceive the goalkeeper.

Corner kick results

One of the most spectacular types of goal that has been scored during a soccer game is that of a direct corner kick. In Fig. 5 one such cases is presented for an initial velocity of 29.04 m/s, an elevation of 21.31 deg and almost pure sidespin of 7.93 rev/s.

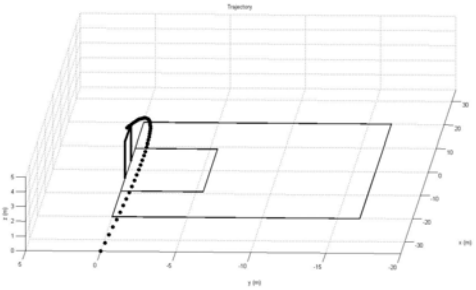

Figure 5: Goal scored by direct corner kick

It is important to note that the appropriate initial conditions were not determined using a cumbersome trial and error method but calculated using a built in Matlab® optimisation algorithm. By incorporating the desired final position of the ball into an appropriate cost function it is possible to calculate the initial velocity and spin vector that will produce the desired result. This technique can obviously also be applied to other game situations.

Goal kick results

In the case of goal kicks the range obtained is of great importance. It is known that for a body only under the influence of gravity the optimum elevation angle for maximum range is 45 degrees. The effect of aerodynamic drag and the Magnus force complicates the dynamics involved.

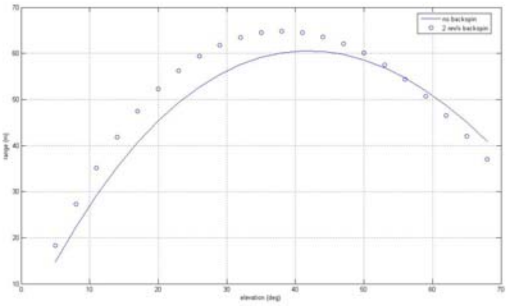

Figure 6: Range versus elevation angle for goal kicks

In Fig. 6 the ranges computed for kicks with no backspin and with 2 rev/s backspin (which due to the upward direction of the generated Magnus force will tend to slightly increase the derived range) are plotted against elevation angle. It is noted that for both cases the optimum angle is below 45 deg and for the case where there is backspin this is significantly lower, at approximately 38 deg.

Discussion of results

The initial results produced from this research work can be used to deduce several interesting points. The free kick results are very similar to those produced by Bray & Kerwin(11). In particular, they too illustrate that 1 deg of difference in the elevation angle can significantly affect the success of the final outcome of the kick and state that their results “illustrate how closely the striker must control the parameters of the kick to achieve successful outcome”. This coincidence with the findings of previous simulation efforts provides confidence that the model is correctly simulating the physical principles governing soccer ball motion in the air.

Top professionals may use several different approaches in order to optimise a free kick result. For example, as stated by Bray & Kerwin(11), there is evidence that several of the top players will strike the ball harder and with a component of topspin. This has the effect of generating a downward component of the Magnus force, thus causing the ball to dip more rapidly. The overall result is for the flight time to be reduced making it more difficult for the goalkeeper to react in time. In Fig. 7, the effect of tilting the spin axis from producing pure backspin, then to sidespin and through to pure topspin, is illustrated. As expected, the largest lateral deflection was caused when pure sidespin in applied. Topspin has a dipping effect on the ball flight (in most cases causing the ball to hit the wall), whereas any amount of backspin causes the ball to travel over the bar.

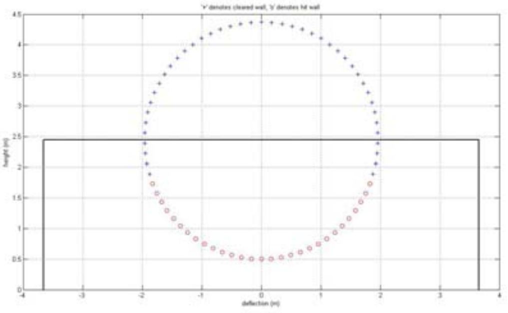

Figure 7: Effect of spin axis tilt

It was not possible to find any simulation data from previous research work regarding the trajectories of corner kicks. From this initial work it is evident that scoring from a direct corner kick is possible but requires great skill in that the ball must be kicked very hard with a substantial amount of spin with a large elevation. It is intended to further this work by investigating alternative corner kick strategies more typical of a normal game situation.

The results regarding the goal kicks are consistent with basic projectile mechanics. Due to the fact that aerodynamic drag will tend to decrease the horizontal component of the velocity, increasing this component by reducing the elevation angle will increase the obtained range. This will result in an optimum angle of less than 45 deg. Adding a small amount of backspin slightly increases the maximum range which is obtained at an angle of approximately 38 deg. Wesson(14), states that approximately 30 deg is the elevation angle usually observed for goal kicks. Although he postulates a different reason for this observed decrease in optimum angle, it could be that the addition of some backspin constitutes a contributing factor.

Finally, weather conditions often play an important roll in the outcome of a soccer game. By adding a component of wind velocity to the ball velocity relative to the air it is possible to approximate the effect of windy conditions on the outcome of a goal kick. In Fig. 8, it is observed that a head wind of 15 m/s will almost halve the range achieved by a goal kick of 30 m/s initial velocity, 38 deg elevation and backspin of 2 rev/s.

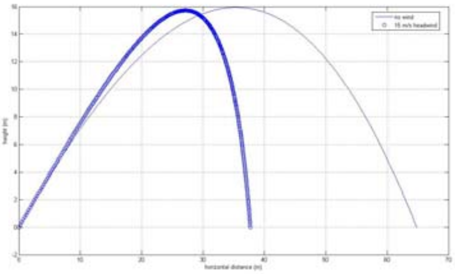

Figure 7: Effect of head wind on goal kick range

Conclusions

- A simple model has been presented that simulates the flight dynamics of a professional soccer ball.
- The aerodynamic characteristics are modelled by inspection of previously published wind tunnel data.
- Initial results are consistent with previous research work and with the physical principles governing soccer ball flight.
- The model can now be used to investigate various situations occurring in the highly popular game of soccer.

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