CALCULATION OF THE FOCAL LENGTH OF A TWO-LENS SYSTEM – AN EDUCATIONAL EXPERIMENT

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Abstract

In the present work, an educational experiment is presented for the estimation of the focal lengths of the individual lenses that compose a two-lens system. The method is based on the measurement of the back focal length and the equivalent focal length of the two-lens system. The experimental setup is simple, two convex lenses of equal focal lengths are positioned at a distance d and by measuring the back focal length of the two-lens system the unknown focal lengths are derived. In addition, the image size observed by the two-lens system is compared to that of different single test lenses in order to derive the equivalent focal length. The experimental methodology is straightforward and provides all the important information that can be obtained for a two-lens system in a geometrical optics laboratory, without expensive equipment.

KEYWORDS: *Educational experiments, optics experiments, lens systems*

1. Introduction

Undoubtedly, the study of lenses and complex optical systems is of great importance for undergraduate courses in surveying, physics, optics and related studies. In recent years there has been renewed interest in developing new educational methodologies and experiments for geometrical optics in undergraduate courses [1-3]. Two-lens systems are widely investigated for both educational and research applications. The focal length of a lens can be easily measured with various methods. The experimental study of a two-lens system can reveal significant details about its individual parts.

In a recent work [4], exact formulas have been presented for thin-lens systems that are made of an arbitrary number of lenses and analytical results have been derived for the focal length of the particular systems. The importance of two-lens systems and its applications is discussed in several Optics textbooks [5-8]. Optical systems used for high image quality photography or other similar applications require optical components and lenses with low aberrations. As a result, designers often use two or more lenses as key elements in a complex optical system in order to minimize aberrations and to obtain higher image quality compared to a single lens solution. Consequently, two-lens systems are frequently used by optical systems manufacturers to provide added value products and the underlying theory has been widely studied since last century and made available to students and undergraduates in the form of excellent textbooks. In the present work, we propose an experimental method which is suitable for undergraduate students' optics laboratories, which allows them to carry out simple measurements in order to calculate the equivalent focal length of a two-lens system and of its component lenses. This experiment extends students' knowledge and can significantly help the educational process due to the simplicity of the experimental apparatus.

2. Methods

Consider a two-lens system composed of thin lenses (1) and (2) of focal lengths f_1 and f_2 respectively that are separated by a distance d, as shown in figure 1. A third lens (Π) is positioned in front of the two-lens system and at a distance from the object light source equal to its focal length, so as to provide collimated rays. These parallel rays pass through the two-lens system and are focused on a screen S. The distance measured from the second lens (2) of the system to the screen S is called back focal length f_B and can be calculated from equation 1 [6].

$$\frac{1}{f_B}=\frac{1}{f_2}+\frac{1}{f_1-d} \quad \text{or} \quad f_B=\frac{f_2(f_1-d)}{f_1+f_2-d} \tag{1}$$

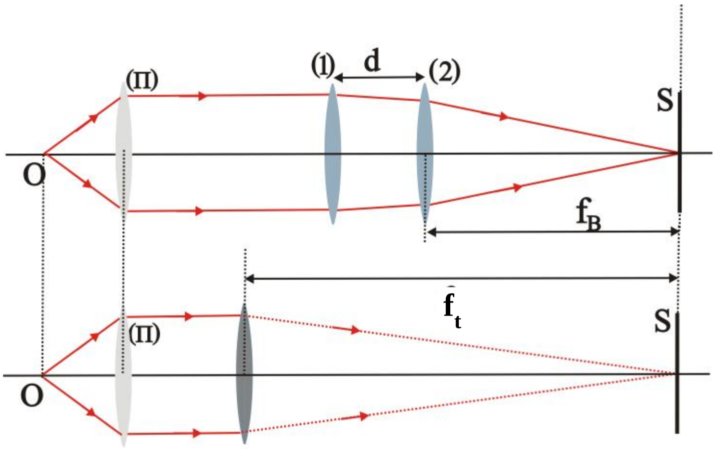

Figure 1: The two-lens system at a distance d is shown at the upper image. The image is formed on a screen S, lens (Π) is used to collimate light from an object O (light source) and f_B is the back focal length of the optical system. A single lens that is equivalent to the two-lens system is shown at the bottom image.

Equation 1 can be derived considering two cases. Initially let's assume that $f_1<d$ as shown in figure 2. Parallel to the central axes rays pass through the first (1) lens and change their direction towards the focal point of the lens F_1. In order to calculate the image distance b_2, the lens equation is applied for lens (2) using F_1 as the object position:

$$\frac{1}{\alpha_2}+\frac{1}{b_2}=\frac{1}{f_2}\Rightarrow\frac{1}{d-f_1}+\frac{1}{b_2}=\frac{1}{f_2} \qquad (2)$$

By definition $b_2=f_B$ and equation 2 can be expressed as:

$$\frac{1}{f_B}=\frac{1}{f_2}+\frac{1}{f_1-d} \qquad (3)$$

In the case of $f_1>d$ (figure 3), the image of the rays passing through lens (1) is formed at F_1, which acts as a virtual object for the second lens, hence by using the lens equation we obtain:

$$-\frac{1}{\alpha_2}+\frac{1}{b_2}=\frac{1}{f_2}\Rightarrow-\frac{1}{f_1-d}+\frac{1}{f_B}=\frac{1}{f_2} \qquad (4)$$

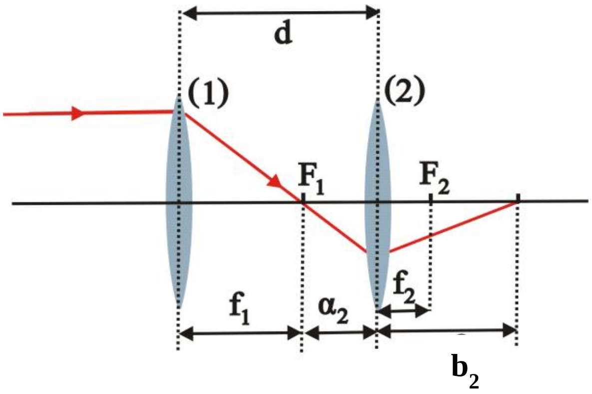

Figure 2: Image formation when the separation distance d between lenses is greater than the focal length f_1 ($f_1<d$).

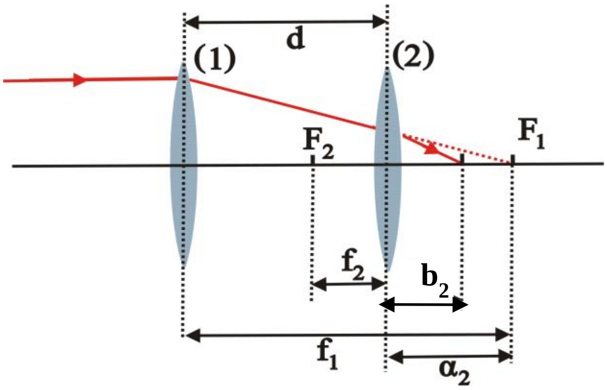

Figure 3: Image formation when the separation distance d between lenses is less than the focal length f_1 ($f_1>d$).

Equations 4 and 3 suggest that the back focal length of the two-lens system can be calculated in any case by equation 1. The mathematics can be simplified by taking two lenses of equal focal length $f_1=f_2=f$, since the purpose of the proposed experiment is educational. By applying equation 1, the focal length of each lens can be expressed as follows:

$$f^2-(d+2f_B)f+f_Bd=0 \Rightarrow f=\frac{(d+2f_B)+\sqrt{d^2+4f_B^2}}{2} \qquad (5)$$

Furthermore, the equivalent focal length f_t of the two-lens system shown in figure 1 can be calculated by equation 6 [6]:

$$f_t = \frac{f_1 f_2}{f_1 + f_2 - d} \tag{6}$$

In the case of equal focal lengths $f_1=f_2=f$ the equation takes the form:

$$f_t = \frac{f^2}{2f - d} \tag{7}$$

3. Experimental setup

In the experimental apparatus a lamp positioned on the left part of the optical bench is used as the object for the two-lens system, as shown in figure 4. A collimating lens (Π) of 15.0 cm focal length is placed between the object and the two-lens system. The distance between lens (Π) and the object is exactly the focal length of the lens in order to collimate the rays before reaching the two-lens system. The two-lens system is made of two lenses (1) and (2) that have equal focal lengths of 15.0 cm.

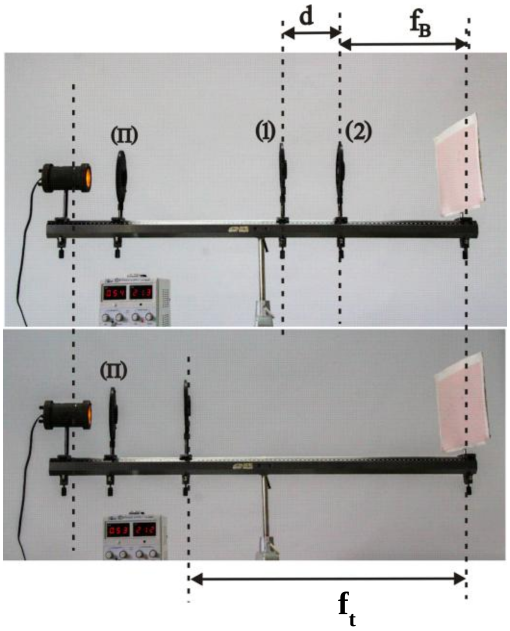

Figure 4: The experimental setup using an optical bench. Upper image shows the two-lens system, lower image shows the single lens setup used for the measurement of the equivalent focal length.

The two lenses are separated by a distance d which can be varied and the system is placed between the collimating lens and the screen. The observation screen is positioned at the right part of the bench, in order to observe the focused image. A single lens setup is shown in the lower photo of figure 4, which has been used for deriving the appropriate image size and thus providing the information about the equivalent focal length of the system.

4. Results

At the first part of the experiment the focal lengths of the individual lenses are measured. The two lenses are positioned at a distance d=10.0 cm. The screen has been moved so that the focused image is clearly formed. The back focal length is measured to be f_B=4.0 cm. Equation (5) is used for the calculation of the focal length f of each lens. By substituting for f_B and d, the derived experimental value for the focal length of each lens is f=15.4 cm.

At the second part of the experiment the equivalent focal length of the two-lens system is calculated. The collimating lens is applied once again. The distance d between the two lenses is changed to a value of d=7.5 cm. The screen is then stabilized at a position close to the right side of the bench where the focused image is clearly formed. The size of the image is measured at this position with a recorded value of l_t=0.5 cm.

Eventually, the two-lens system is removed from the optical bench, without moving the screen. In order to experimentally estimate the equivalent focal length of the two-lens system, three convex lenses of different focal lengths have been utilized as test lenses. Each test lens is positioned after the collimating lens as shown in figure 4 (bottom image) and the respective image size is estimated on the stable screen. The experimental results are shown in table 1, where the image size l is recorded for each test lens.

Table 1: The focal length of the single test lenses used and their corresponding image size

focal length (cm)	l (cm)
5.0	0.2
10.0	0.5

15.0	0.7

5. Discussion

At the first part of the experiment the two-lens system has been used for the calculation of the focal length of the individual lens. The calculations are straightforward and easily understood by the students due to the equal focal lengths of the individual lenses. The experimental value of the focal length is estimated as f=15.4 cm, in good agreement with the catalogue value of the lens used, which is 15.0 cm. This is equivalent to a 2.7% variation from the catalogue value, which is an excellent result for a students' laboratory. Consequently, the first part of the experiment provides an easy methodology for deriving information about the single lenses just by measuring the back focal length of the two-lens system, which can be employed in any optics laboratory without expensive equipment.

At the second part of the experiment the equivalent focal length of the two-lens system has been calculated with the help of different individual test lenses. The image size of the two-lens system can be compared to the one observed using each test lens. As shown in table 1, a single test lens of 10.0 cm focal length generates the same image size with the two-lens system. Consequently, the equivalent focal length of the two-lens system is approximately equal to f_t=10.0 cm. This measurement can be compared to the theoretical value derived from equation 7, by substituting the values of f=15.0 cm and d=7.5 cm used in our case. The theoretical value is f_t=10.0 cm, which is in excellent agreement with the experimentally estimated value.

6. Conclusions

The study of a two-lens system can provide important information about the individual parts that compose the optical system. A two-lens system of equal focal lengths is a simple and excellent introductory experiment for undergraduate students. Initially the focal length of the single lenses that compose the system can be estimated just by measuring the back focal length on an optical bench. In addition, the equivalent focal length of the two-lens system is determined by comparing the image sizes of the two-lens system to the image sizes of individual test lenses. Both parts lead to experimental results that are in good agreement with the theoretically calculated values. The simplicity of the experiment and the straightforward measurement procedure make the proposed experimental apparatus an important laboratory tool that can awaken the motivation of students to study more complex issues of geometrical optics.

7. References

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